



Wantirna College

Place student
identification sticker
here

MATHEMATICAL METHODS CAS UNIT 1

TECHNOLOGY FREE EXAM Written examination

Wednesday 6th June 2012

Reading time: (10 minutes)

Writing time: (60 minutes)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
12	12	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied

- Question and answer book of 6 pages.
- Additional paper is available if needed to complete an answer.

Instructions

- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Teacher's initials : AUM BEA TEB circle correct one

Linear Functions

$$y = mx + c$$

$$m = \frac{\text{rise}}{\text{run}} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$m_1 \times m_2 = -1$$

Quadratic Functions

$$\text{Quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Discriminant: } \Delta = b^2 - 4ac$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Circular Functions

$$\text{To convert degrees to radians multiply by } \frac{\pi}{180}$$

$$\text{To convert radians to degrees multiply by } \frac{180}{\pi}$$

$$\text{For } y = \sin nx \text{ and } y = \cos nx \text{ the period is given } \frac{2\pi}{n}$$

$$\text{For } y = \tan nx \text{ the period is given } \frac{\pi}{n}$$

$$\text{When graphing } y = \tan nx \text{ the spacing of the asymptotes is } \frac{\pi}{n}, \text{ asymptotes occur at } \frac{\pi}{2n} \text{ and } \frac{3\pi}{2n}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

1. Find the value of the discriminant and hence determine the number and type of solutions for:

$$-4x^2 - 3x + 2 = 0$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-3)^2 - 4 \times -4 \times 2 \\ &= 9 + 32 \\ &= 41\end{aligned}$$

$$\begin{aligned}x &= \frac{-(-3) \pm \sqrt{41}}{2 \times -4} \\ &= \frac{3 \pm \sqrt{41}}{-8}\end{aligned}$$

(3 marks)

2. Factorise fully: $2x^2 + 16x + 32$

$$\begin{aligned}2(x^2 + 8x + 16) \\ 2(x + 4)^2\end{aligned}$$

(2 marks)

3. a) The midpoint C of the line segment joining A (0, 6) to B has coordinates (-3, -1). Find the coordinates of B.

$$\begin{aligned}\text{Midpt} &= (-3, -1) = \left(\frac{0 + x_B}{2}, \frac{6 + y_B}{2} \right) \\ -3 &= \frac{x_B}{2} & -1 &= \frac{6 + y_B}{2} \\ x_B &= -6 & -2 &= 6 + y_B \\ & & y_B &= -8\end{aligned}$$

B (-6, -8)

- b) Find the distance AC

$$\begin{aligned}AC &= \sqrt{(0 - (-3))^2 + (6 - (-1))^2} \\ &= \sqrt{3^2 + 7^2} \\ &= \sqrt{9 + 49} = \sqrt{58} \text{ units}\end{aligned}$$

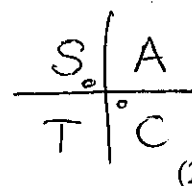
(2+2 marks)

4. Find the value(s) of x if $\tan x = \frac{-1}{\sqrt{3}}$ given $0 \leq x \leq 2\pi$

base angle

$$x = \frac{\pi}{6} \text{ in } (2, 4)$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$



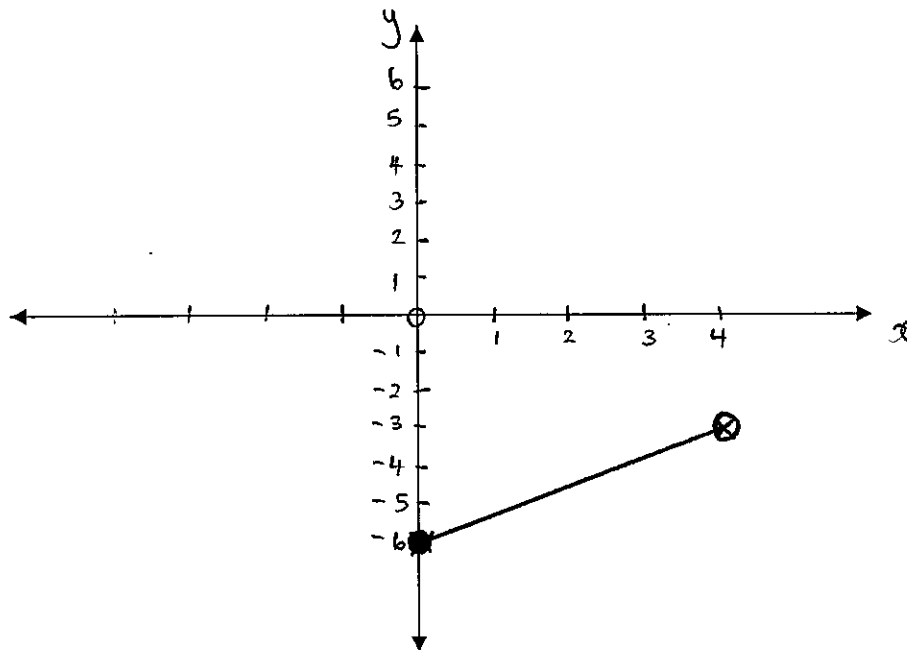
(2marks)

$$x=0 \quad -4y=24 \\ y=-6$$

$$y=0 \quad 3x=24 \\ x=8$$

$$x=4 \\ -4y+12-24=0 \\ -4y=12 \\ y=-3$$

- 5 a) Sketch the graph of: $-4y+3x-24=0$, where $0 \leq x < 4$



- b) What is the range of the function?

$$\text{ran} \in [-6, -3)$$

(3+1=4 marks)

6. Solve the following two equations simultaneously

$$y = -3x + 3$$

$$3y + 2x = 2$$

$$3(-3x + 3) + 2x = 2$$

$$-9x + 9 + 2x = 2$$

$$-7x = -7$$

$$x = 1$$

(3 marks)

7. $P(x) = ax^3 + 2x^2 - 3$ and $Q(x) = 2x^2 + bx + c$ and $P(x) \equiv Q(x)$ find a , b and c .

$$ax^3 + 2x^2 + \boxed{0x} - 3 \equiv \boxed{0x^3} + 2x^2 + bx + c$$

$$a = 0$$

$$b = 0$$

$$c = -3$$

(3 marks)

8 (a) Solve for x : $9(x+1)^2 - 36 = 0$

$$9(x+1)^2 = 36$$

$$(x+1)^2 = 4$$

$$(x+1) = \pm 2$$

$$x = \pm 2 - 1$$

$$x = -3, 1$$

$$tp = -1, -36$$

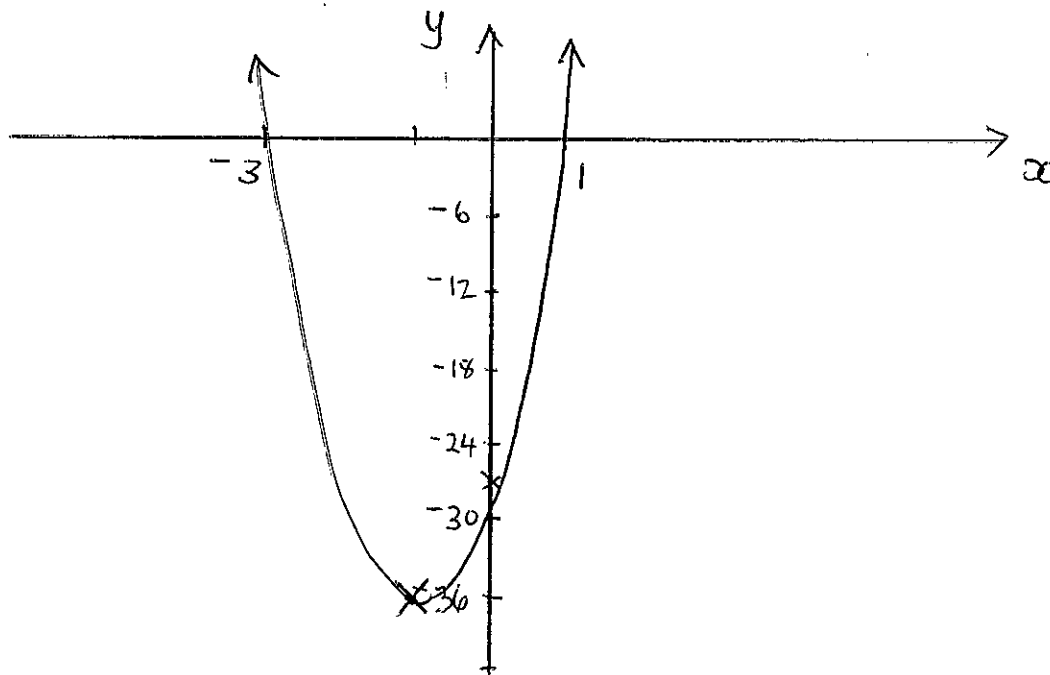
$$y \text{ int } x = 0$$

$$9 - 36 = y$$

$$-27$$

(2 marks)

(b) Using your answer to part (a), sketch the graph of $y = 9(x+1)^2 - 36$, clearly showing the coordinates of all relevant points



(4 marks)

9. Express $y = x^2 + 4x + 10$ in the form $y = a(x-h)^2 + k$

$$y = (x+2)^2 - 4 + 10 \quad a = 1$$

$$= (x+2)^2 + 6 \quad h = -2$$

$$\quad \quad \quad k = 6$$

(2 marks)

10. A cosine graph has a maximum value of 12 and a minimum value of -2. Its period is 4. Find a possible equation.

(3 marks)

11. a) Change 330° to radians in terms of π

$$\frac{11\pi}{6}$$

$$\left(\frac{330^\circ}{180^\circ} = \frac{330\cancel{\pi}}{180\cancel{\pi}} \right) 1^\circ = \frac{\pi}{180^\circ}$$

b) find the exact value of $\sin 330^\circ$

(3 marks)

$$\text{base angle } 30^\circ \quad \sin 330^\circ = -\sin 30$$

$$= -\frac{1}{2}$$

4th Q

$$\frac{S}{T} \mid \frac{A}{C}$$

Not yet covered

Not yet
covered

12. A fisherman finds that the height of the tide in the harbour can be found using the equation:

$$h(t) = 5 - 2 \sin\left(\frac{\pi t}{15}\right)$$

where h metres is the height of the tide and
 t is the number of hours after midnight.

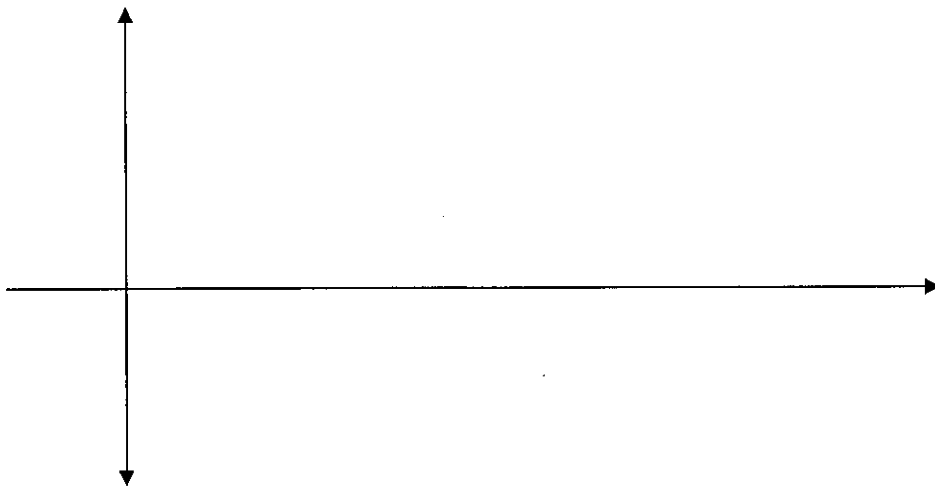
- (a) What is the height of high tide?

(1 mark)

- (b) What is the period of this function?

(1 mark)

- (b) Sketch one complete period of the graph of $h(t)$.



(3 marks)